Circular Polarization of the Magnetic Field in the WG Modes of Resonance of a Dielectric Disc at Microwave Frequencies

Iginio Longo, Pierre Guillon, *Senior Member, IEEE*, and Dominique Cros

***Abstract*— The circular polarization of the magnetic field of the Whispering-Gallery (WG) modes of resonance of a dielectric disc resonator has been tested in the evanescent-field region outside the dielectric material. Microwaves in the frequency range from 18 to 26.5 GHz (K-band) and the techniques of Electron Spin Resonance (ESR) have been used. The distribution of the electromagnetic fields was obtained with a finite element method and it was found in agreement with the experimental results.**

I. Introduction

Whispering-gallery (WG) dielectric resonators have been the object of many investigations, in view of their applications as elements of microwave integrated circuits [1]. These resonators can be acting on their modes which are classifyed as either $\mathrm{WGE}_{n,m,l}$ or $\mathrm{WGH}_{n,m,l}$, the integer numbers n, m and l denoting respectively the azimuthal, radial and axial field variations. In the resonant modes of the first family, (WGE modes), the electric field is essentially radial while it is essentially axial for the second ones. Since the WG modes of resonance are of the travelling wave type (ring modes), a fourth index, indicating the clockwise or the counter-clockwise direction of propagation of the microwaves would be necessary to denote more completely the mode. WG dielectric resonators at microwave frequencies are usually disc shaped, of low aspect ratio, made out of low loss and high dielectric constant materials. The travelling wave resonant WG mode is confined between the curved surface of the disc, and a fictitious internal one, called "caustic surface," whose radius depends on the mode, on the sizes and on the dielectric constant of the resonator material. An evanescent field is present outside the resonator, decaying exponentially. Due to the very low radiation losses, WG dielectric resonators can be operated without a metal enclosure.

In the domain of RF spectroscopy, an application of these special resonators to the techniques of Electron Spin Resonance (ESR) has been proposed [2], [3]. In this paper we show that the ESR techniques can be applied to test the circular polarization of the magnetic field of some WGH modes of resonance.

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II. The Method

Since the adopted experimental method makes use of the ESR techniques, it will be useful to recall briefly that the traditional ESR spectrometer consists essentially in a microwave source-and-waveguide system feeding a metal cavity resonator. A paramagnetic sample is placed inside the cavity at a point of high microwave magnetic field intensity. A static magnetic field $\boldsymbol{H}_0$ perpendicular to the cavity microwave magnetic field is applied to the sample. The microwave power reflected or transmitted by the cavity resonator, at its resonant frequency, is recorded while the static magnetic field is slowly swept across an absorption line. In an ESR experiment the unpaired spins of the paramagnetic sample absorb energy when they interact with a circularly polarized microwave field $\boldsymbol{H}_1 \sin \omega t$, having the same frequency and the same sense of the spin state's precessional motion in the field $\boldsymbol{H}_0$ [4]. The resonance condition is given by

$$\omega = \gamma \boldsymbol{H}_0 \tag{1}$$

γ being the gyromagnetic factor. In case of free electrons one has $\gamma = 17.6 * 10^{10}(\sec \mathrm{T})^{-1}$. In a static magnetic field H_0 of approximately 0.72 T, the paramagnetic resonance occurs at microwave frequencies in the K-band.

In almost all ESR experiments the microwave magnetic field $\boldsymbol{H}_1 \sin \omega t$ inside the cavity resonator is linearly polarized. It can be decomposed into two counter-rotating circularly polarized components, only one of which induces transitions, the other being ineffective. If one reverses the direction of the static magnetic field, according to (1), the role of the two components is exchanged, so that the resulting ESR signal amplitude is not affected.

Let us consider a paramagnetic point sample submitted to the magnetic field of a travelling-wave, such as the propagating $\mathrm{TE}_{1,0}$ mode of a standard metal waveguide of rectangular cross section. Let A, B, C and C′ be the various positions of the sample, as in Fig. 1(a). In this figure the instantaneous magnetic field lines in the H plane of the waveguide are drawn, the propagation occurring from left to right. In the positions indicated by A and B the sample is submitted to a linearly polarized magnetic field, so that, in a rectangular reference frame whose x and y axis are parallel to the waveguide sizes and the z axis is directed along the direction of propagation, only the z or the x components of the microwave magnetic field is present, respectively. In the

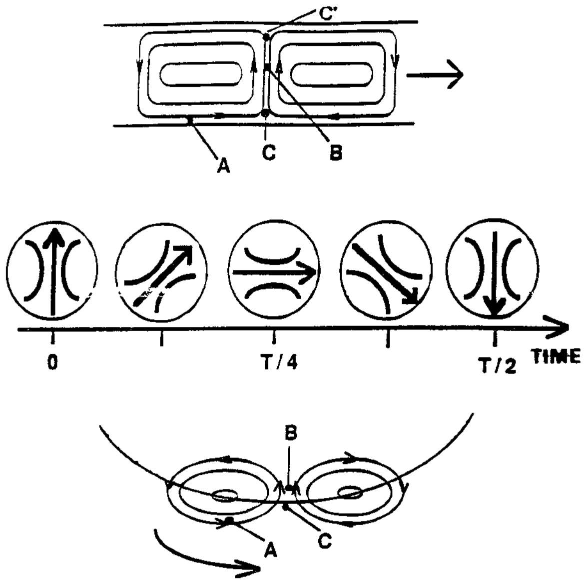

Fig. 1. (a) The magnetic field lines in the H plane of a standard rectangular metal waveguide propagating the $TE_{1,0}$ mode. A, B, C, and C′ are possible sample positions in an ESR experiment in which the static magnetic field is perpendicular to the plane of the drawing. (b) The polarization of the travelling magnetic field as observed by a point sample in C, during half a period of the wave. The field looks circularly (clockwise) polarized. (c) The magnetic field lines of a $WGH_{n,0,0}$ mode propagating along the periphery of a dielectric disc resonator.

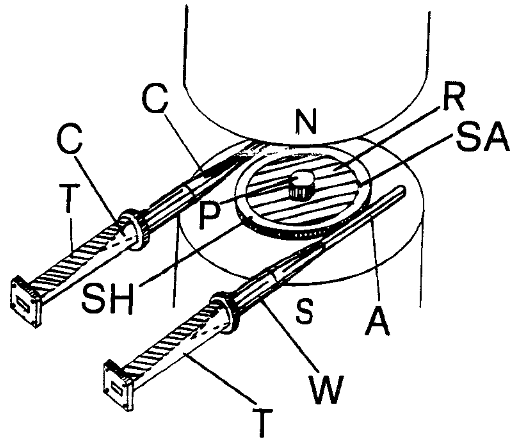

Fig. 2. A view of the dielectric resonator and the coupled waveguides in a ESR experiment. T: rectangular-to-circular standard waveguide transition. W: 8 mm fused quartz waveguide. A: 5 mm dielectric rod absorber. C: conical taper. R: dielectric disc resonator. SA: paramagnetic point sample. SH: PTF sample-holder. P: non magnetic post. N, S: electromagnetic pole-pieces.

particular position C however the sample sees a clockwise circularly polarized field, whose successive orientations during a semiperiod are shown in the five insets in Fig. 1(b). At the position indicated by C′ the sense of the circular polarization is reversed. In the C and C′ positions the H_x and H_z components have equal amplitudes.

Let us calculate the position C in a practical case. In a standard K-band waveguide of $4 \times 10\,\mathrm{mm}^2$ cross section and operating at a guide wavelength λ_g of 15 mm, the C and C′ positions are about 2 mm of distance from the small size of the waveguide.

The same conclusions can be drawn considering the propagation of a $WGH_{n,0,0}$ mode along the periphery of a dielectric disc resonator. Fig. 1(c) shows a pictorial view of the magnetic field lines in an equatorial plane of the dielectric resonator. Thus the magnetic field lines of a $WGH_{n,0,0}$, in the plane of the resonator, as depicted also by Jiao *et al.* [1], are looking much like those of the $TE_{1,0}$ mode of the standard rectangular metal waveguide in the H plane, so that experiments using the circular polarization of the travelling magnetic field near the disk rim can be done.

ESR experiments with a point sample can be useful to get informations on the polarization of the microwave magnetic field in the plane of the disc.

III. Experimental Layout

In our experiments we used a high-purity synthered alumina disc-shaped resonator. The diameter was 20 mm and the thickness 3 mm. The disc had a central hole of 6 mm, useful for handling.

In this work, in the K-band, the $WGH_{n,0,0}$ modes, of resonance, having the H field lines mostly in the plane of the disc, were excited by the evanescent field of a 8 mm fused quartz rod W, as depicted in Fig. 2. The transmitted power was detected using an identical dielectric waveguide, terminated by a matched crystal detector or by a microwave power meter. The dielectric rod waveguides were connected to a standard rectangular metal waveguide through a rectangular to circular waveguide transition. The conical tapers C at the ends of the dielectric rods were useful for impedance matching purposes. The tapers were about 3 cm long. A 5 mm fused quartz rod, 5 cm in length, was welded to one end of the tapers of the dielectric waveguides and painted with colloidal graphite. These 5 mm dielectric rod absorbers reduced both the unwanted microwave reflections and the outward radiated power. This excitation scheme gave a good directional character to the coupling between the resonator and the waveguide. Directivities larger than 20 dB were obtained by properly positioning the dielectric components. The WG resonator R was held between the polar expansions of a small electromagnet, with pole-pieces diameter of 7 cm, using a dielectric post P passing through its central hole. Various $WGH_{n,0,0}$ modes, with $7 < n < 12$ were excited using a HP model 3508A microwave sweep source with a 83570 RF plug-in for the K-band operations.

An ESR experiment was accomplished by placing the disc resonator, loaded with a point DPPH sample (diphenyl-picryl-hydrazyl free radical) and excited in a $WGH_{n,0,0}$ mode of resonance, between the pole-pieces of the electromagnet. The static magnetic field was perpendicular to the plane of the disk and hence to the microwave magnetic field.

The point sample was applied onto the curved surface of the resonator rim. In a set of experiments a PTF sample-holder SH, as depicted in Fig. 2, having the shape of a ring of $3 \times 3\mathrm{mm}^2$ square cross-section, was placed around the disk resonator and used to hold about 500 μg of DPPH. In this case the sample was placed inside a hole drilled near the internal rim of the PTF ring, the hole axis being parallel to the disk axis. The

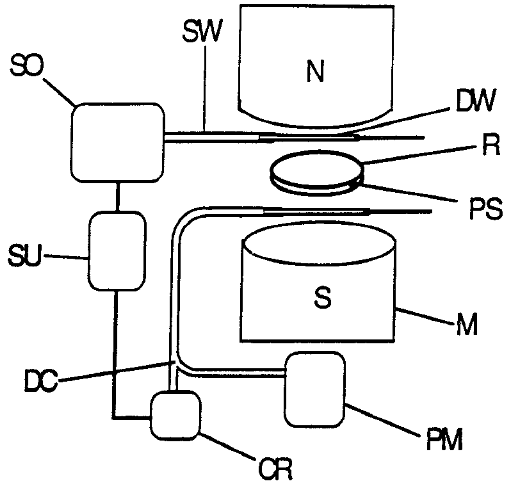

Fig. 3. Experimental set-up. SO: sweep oscillator. SU: stabilizing unit. DC: directional coupler. CR: crystal receiver. PM: microwave power meter. M: electromagnet pole-cap. R: disc resonator. PS: point sample glued onto the resonator curved surface. SW: standard K-band waveguide. DW: dielectric waveguide.

hole was about 1.8 mm deep and its diameter was of 0.8 mm. The point sample was nearly touching the alumina disk, the clearance being of about 0.5 mm.

In another experiment the same amount of polycrystalline DPPH was mixed with a low-loss epoxy glue. The black sample was glued to the curved rim of the resonator (in the middle of its thickness) and had a nearly semispherical shape of 1 mm of radius, as depicted in Fig. 3.

IV. Results

We used the standard methods of ESR spectroscopy, having mounted the resonator in a "transmission" configuration, as depicted in Fig. 3. The microwave source was stabilized at the frequency of resonance of the dielectric disc. The static magnetic field was slowly swept across the narrow DPPH absorption line ($\Delta\nu = 2 * 10^{-4}$T), and the peak signal was recorded. Due to the high sensitivity attainable with this method, it was not necessary to use lock-in detection. A signal to noise ratio in excess of 15 was easily obtained observing the ESR signals directly on the oscilloscope, without any particular noise reduction device. Peak voltage signals in the 100 mV range were obtained with a crystal detector, using a 1N26 receiver and a 10 dBm microwave source.

The percentage of circular polarization was obtained this way: we measured, at each resonant frequency of the disc resonator, the EPR peak power signal S_D. Then, after having inverted the static magnetic field, the peak power signal S_I was measured.

Since the power signal in a ESR experiment, in absence of saturation phenomena, is proportional to the square of the microwave magnetic field amplitude acting on the sample, the $(S_D/S_I)^{1/2}$ ratios give indications on the relative amplitudes of the components of magnetic field of the microwave travelling wave, at the same position of the point sample S, in the plane of the disc. These ratios do not depend on the Q factor of each mode of resonance, nor on its resonant frequency, nor on the disc material.

When this ratio is equal to 1, the microwave magnetic field is linearly polarized, so that only one component of the magnetic field is present, namely the radial one. When the polarization is exactly circular, the ratio should diverge to infinity, because one of the signals disappears ($S_I = 0$), in accordance with the elementary theory of the ESR signals. In this case the radial and the azimuthal components have equal amplitudes.

In general the magnetic field of the WGH travelling wave, as it is seen by the paramagnetic point sample, is elliptically polarized, and can be thought of as the sum of a circularly polarized component which gives the larger S_D signal, and a counter-rotating one (of lesser amplitude) which gives the smaller signal S_I, when the static magnetic field is inverted.

The square root $\rho = (S_D/S_I)^{1/2}(\rho > 1)$ represents how many times the useful circularly polarized component of the wave is larger than the counter rotating one, and hence the percentage of circular polarization in a ESR experiment is given by the relation:

$$c = 100 * (\rho - 1)/\rho. \quad (2)$$

During the experiments a particular attention had to be paid to the measurements performed on nearly degenerate modes. Mode degeneracy may occur for two particular set of indices such as $(n, 0, 0)$ and $(n', m, 0)$. This degeneracy, if any, could be noted by observing the resonance tip of each mode on the oscilloscope, while the dielectric waveguides position is slightly changed. The tip broadens and eventually the mode of resonance splits into two partially resolved ones. Another possible cause of errors was the partial mode splitting when loading the resonator with a too large sample. In the presentation of the experimental results we have been more confident on single-mode operations, and moreover we used a sample mass as small as possible to obtain video detection of the ESR signal without mode splitting.

The Q-factors were measured using a microwave synchronizer to obtain accurate readings of each resonant frequency and of its width at half height. The Q factors were obtained by measuring directly the central frequency of the resonant transmitted curve and the frequencies of its half-height points, keeping the resonator loosely coupled to the dielectric rods, without the paramagnetic sample, and using a stabilized microwave source. The Q-factors were measured without the static magnetic field acting on the dielectric resonator, so that the absorption due to paramagnetic centers included into the alumina material or in the PTF holder did not play any role. Table I shows the calculated and the measured resonant frequencies of the $\mathrm{WGH}_{n,0,0}$ modes of the alumina disk and the loaded Q factors, in absence of the static magnetic field. Both the resonant frequencies and the Q-factors of each resonant WGH mode were affected by the absorption of microwaves at the paramagnetic resonance, as when one utilizes a conventional metal cavity resonator.

The measurements of S_D/S_I were dependent to some extent on the coupling between the WG resonator and the dielectric waveguides. Stationary waves are easily excited. The coupling

TABLE I
MEASURED AND CALCULATED RESONANT FREQUENCIES AND LOADED Q-FACTORS OF THE VARIOUS $WGH_{n,0,0}$ MODES OF RESONANCE OF THE TRAVELLING WAVE MAGNETIC FIELD IN THE PLANE OF THE DISC RESONATOR

Frequencies (GHz) (meas.)	Frequencies (GHz) (calc.)	Modal Indices	Q-Factor
22.06	22.08	8,0,0	1950
23.49	23.44	9,0,0	4700
24.84	24.44	10,0,0	8200
26.23	26.28	11,0,0	8700

was accurately adjusted for each excited mode of resonance in order to maximize the S_D/S_I ratio, and was not changed during the inversion of the static magnetic field. A weak coupling was necessary in order to prevent stationary waves which can affect the measured ratios up to a factor of 5.

When the resonator was operated without the DPPH sample, the signal base line on the oscilloscope, corresponding to a sweep of the static magnetic field near the DPPH line, showed a rather broad absorption feature whose peak voltage resulted to depend on the orientation of $\boldsymbol{H}_0$. This non symmetric absorption signal had a resonance width of about $2 * 10^{-3}$ T. Its width however was attributed to the inhomogeneities of the static field acting in this case upon a 20 mm sample (the resonator itself). There are many possible causes of this effect, the most likely are the inhomogeneities of the distribution of paramagnetic centres and/or of the magnetic fields inside the dielectric material. However the measurements showed that this effect was of a few per cent of the ESR peak signal of the DPPH sample. In some cases, when the S_I signal was weak, it appeared on the oscilloscope showing a first derivative lineshape, because of the inclined base line. This was another cause of errors which we had to account for. The completely open structure of the dielectric resonator and waveguide probehead did not permit accurate measurements of the ESR peak signals, especially when measuring S_I, due to the fact that the unavoidable radiation of microwaves from the exciting dielectric rod into the free space induced fluctuations in the power received by the other dielectric waveguide.

The results obtained with the resonator having the sample glued onto its curved surface and accounting for the percentage of circular polarization as given by (2) are reported in Fig. 4. Error bars represent the largest deviations observed during experiments utilizing three different resonators of the same sizes. The experimental points show that just outside the resonator curved rim, the $WGH_{9,0,0}$, $WGH_{10,0,0}$ and $WGH_{11,0,0}$ modes of resonance have an evanescent magnetic field with more than 75% of circular polarization [5]. The experiments performed with the PTF sample holder furnished analog results. In this case the resonant modes with lower azimuthal index in some cases had an even larger percentage of circular polarization. It is worth pointing out that the application of the PTF ring did modify slightly the extent of the exponential decay behavior of the evanescent field outside the resonator, affecting the resonant frequencies by a few per cent.

In another set of experiments we excited the same WGH modes, but travelling in the opposite direction. This was obtained by exchanging the role of the two dielectric rod waveguides. We found that the same results in ESR experiments were obtained simply by reverting the static magnetic field. These experiments demonstrated clearly the travelling wave character of the excited WGH modes. The independence of the ESR signals on the azimuthal sample position was also exemplified, within the experimental errors, provided that the sample was of small size, so that stationary waves were not excited.

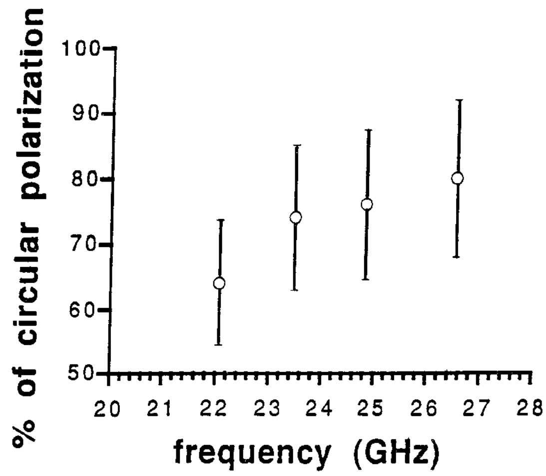

Fig. 4. The percentage of circularly polarized microwave magnetic field in the plane of the disc resonator, at various modes of resonance.

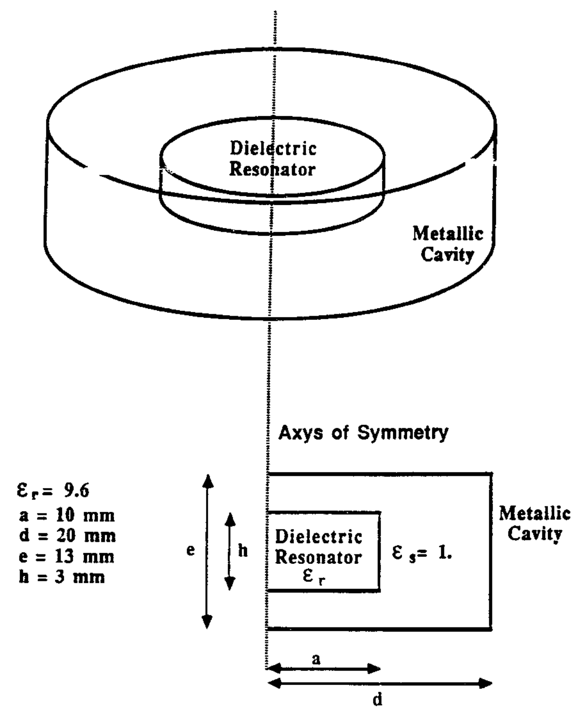

Fig. 5. The dimensions utilized for the calculations of the resonant frequencies.

Calculations were performed in order to confirm the experimental results. To define analytically the electromagnetic field components it is necessary to use a very sophisticated analysis. We have chosen a numerical approximate solution, as given by the finite element method. The finite element

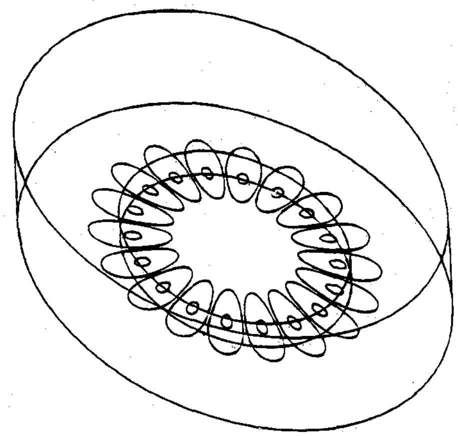

Fig. 6. Computed display of the magnetic field lines of the $WGH_{9,0,0}$ mode of a resonator placed inside a metal cavity.

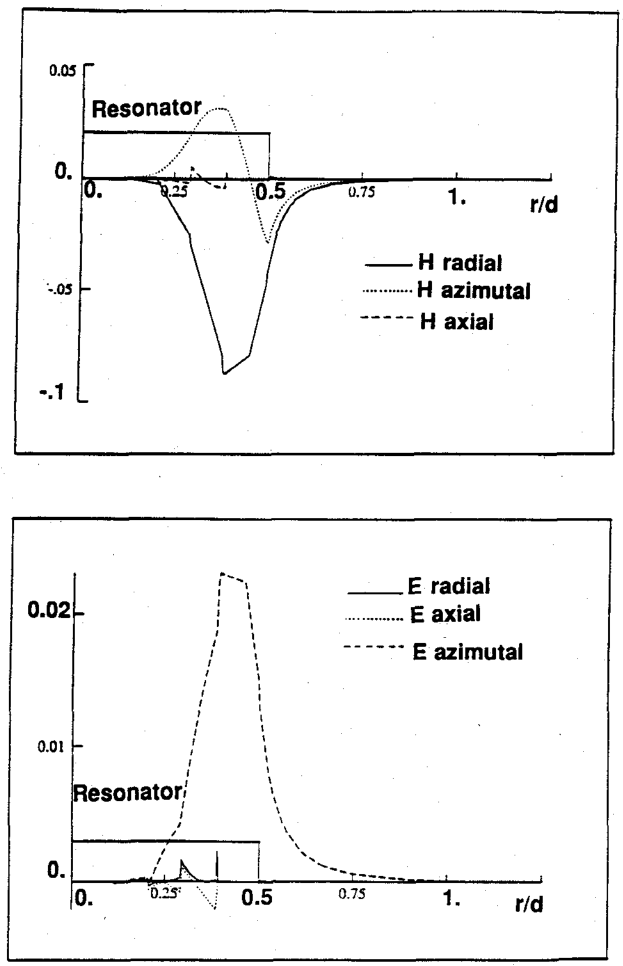

Fig. 7. The calculated magnetic and electric fields components of the $WGH_{9,0,0}$. The field amplitudes, in arbitrary units, are drawn as a function of the ratio r/d, where r is the radial coordinate and the cavity radius.

method used here utilizes a triangular map of the resonator volume, and it is associated with the Nedelec polynomials of second degree [6]. A cylindrical metal cavity surrounding the dielectric disc, as depicted in Fig. 5 was introduced to give the Maxwell equations suitable boundary conditions. The numerical analysis permits the calculation of the resonant frequencies and also a graphical display of the field associated with each resonant mode.

The magnetic field lines and the field display of the $WGH_{9,0,0}$ mode, as an example of the resolution attainable, are reported in Figs. 6 and 7, respectively. The calculated resonant frequencies are in good agreement with the measured ones. The radial and the azimuthal microwave magnetic field components of the $WGH_{9,0,0}$ at the disc periphery, are about in the 3 : 4 ratio, giving a circular polarization of 75%, also in very good agreement with the experimental data. The calculation concerning the resonant modes with n = 8, 9, 10, 11 demonstrate that the radial and the azimuthal magnetic field component, in the evanescent field region just outside the disc rim, are becoming of equal magnitude as the n index increases.

V. Conclusions

An experimental investigation of the polarization of the magnetic field in a travelling-wave dielectric disc resonator of syntered alumina, 20 mm of diameter and 3 mm thick, excited in the $WGH_{n,0,0}$ modes of resonance in the K-frequency band, has been carried out using a new ESR technique. Numerical calculations utilizing the finite element method confirm the results. A 75% of circular polarization of the magnetic field in the evanescent field region just outside the dielectric resonator has been obtained. The measured resonant frequencies of the $WGH_{n,0,0}$ modes and the relative amplitude of the microwave magnetic field are in good agreement with the numerical calculations. The results obtained with the application of the ESR techniques can be used for the design of non-reciprocal components of microwave integrated circuits.

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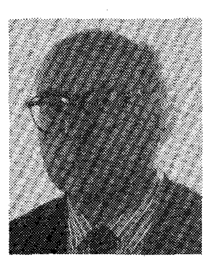

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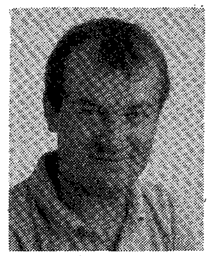

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